Lecture 12 - February 24

Model Checking

Operator Precedence
Parse Trees, LMDs, RMDs

Announcements/Reminders

- **ProgTest1** grading started on SUN, Feb 23
  + Expected to get raw results from TAs by MON, Mar 3
- **Lab3** to be released
- **WrittenTest1** guide to be released
- This week's office hour: 3pm, Wed
- TA contact information (on-demand for labs) on eClass

Operator Precedence

(...)

In the absence of pathentheses, what's the order of evaluation?

tightest

/* Unary Temporal Operator */

X, F, G

next, future, global

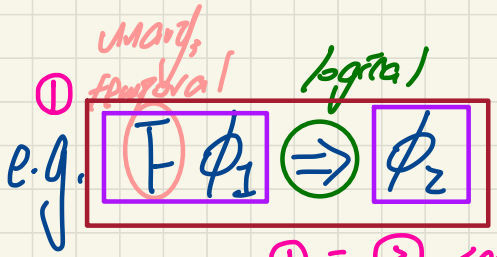
/* Binary Temporal Operator */

U, W, R

until, weak until, release

/* Logical Operator */

$\neg$, $\wedge$, $\vee$, $\Rightarrow$



① ≡ ② same meaning

② ≠ ③ not semantically equivalent.

vs.

② $(F\,\phi_1) \Rightarrow \phi_2$

vs.

③ $F(\phi_1 \Rightarrow \phi_2)$

Letter	symbol
x	○
F	◇
G	□

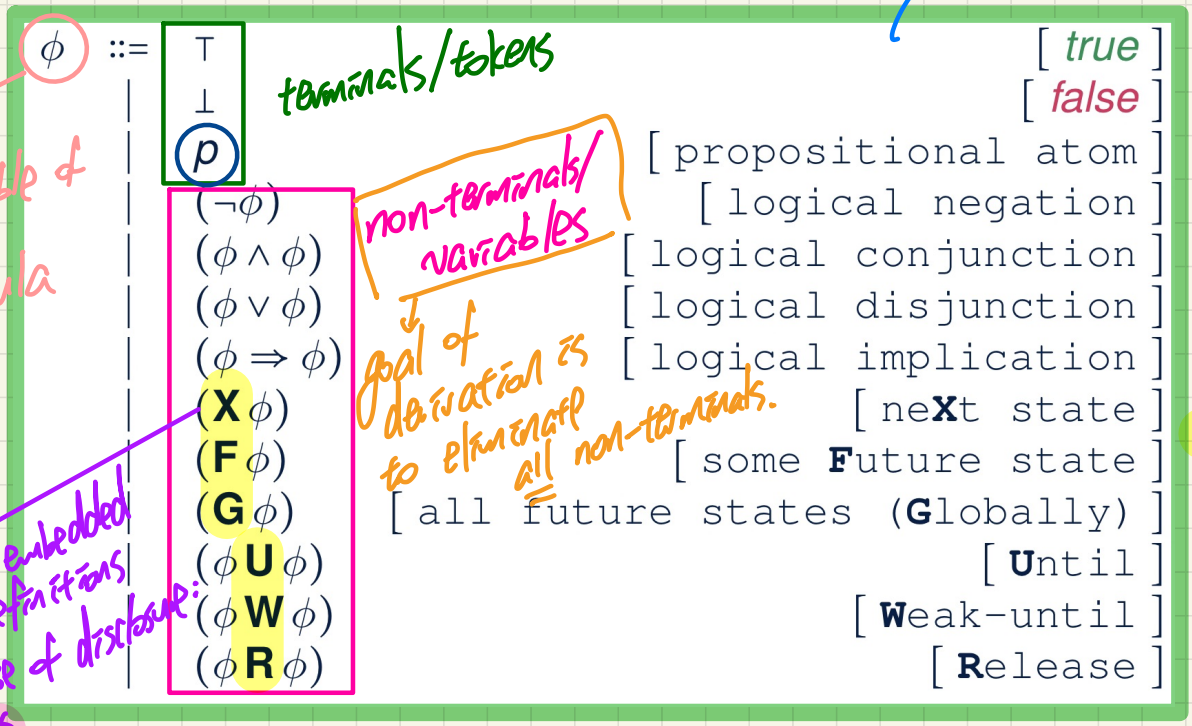
(1) $\dfrac{x\,\phi}{\text{III}}$

○ϕ

(2) $\dfrac{F\,G\,\phi}{\text{III}}$

◇□ϕ

Parsing: Some Practical Knowledge



Assumption: Operator **precedence** considered first before the CFG.

Interpreting a Formula: Parse Trees (1)

$$\phi ::= \top \mid \bot \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \Rightarrow \phi) \mid (\mathbf{X}\phi) \mid (\mathbf{F}\phi) \mid (\mathbf{G}\phi) \mid (\phi\,\mathbf{U}\,\phi) \mid (\phi\,\mathbf{W}\,\phi) \mid (\phi\,\mathbf{R}\,\phi)$$

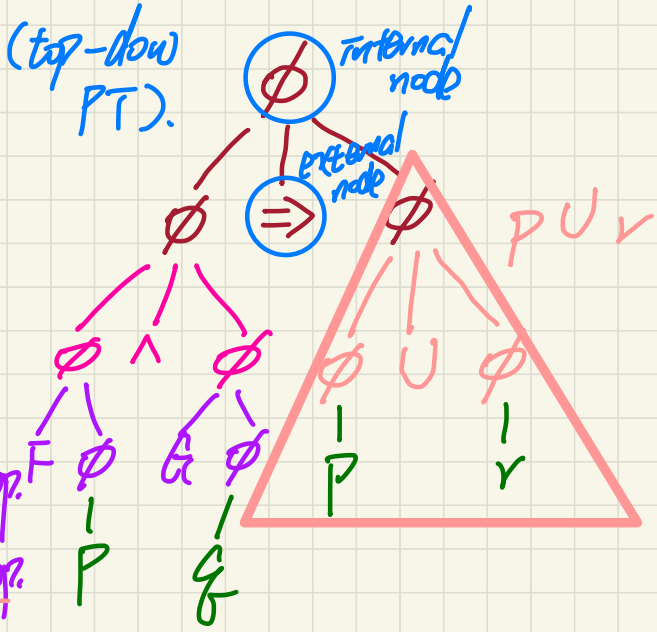
[true] [false] [propositional atom] [logical negation] [logical conjunction] [logical disjunction] [logical implication] [neXt state] [some Future state] [all future states (Globally)] [Until] [Weak-until] [Release]

not a keyword

Can be: p, q, r, ... (>0)

$$\mathbf{F}\,p \wedge \mathbf{G}\,q \Rightarrow p\,\mathbf{U}\,r$$

(top-down PT).



Before considering the CFG, first use operator precedence to group terms

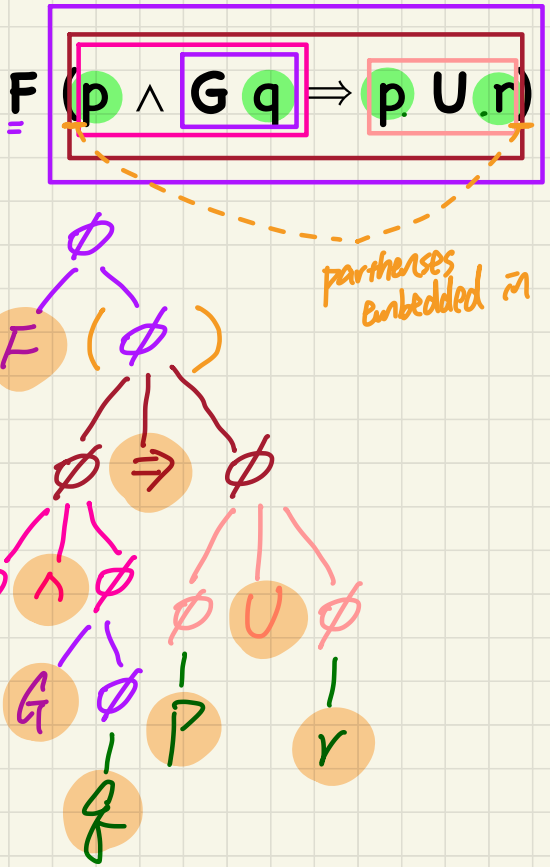
Unary Temp. op.

Binary Temp. op.

Logical op.

Interpreting a Formula: Parse Trees (2)

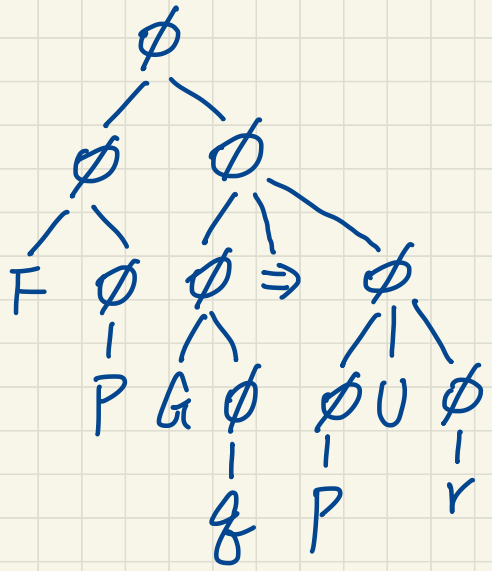
ϕ	::=	$\top$	[*true*]
	|	$\bot$	[*false*]
	|	p	[propositional atom]
	|	$(\neg\phi)$	[logical negation]
	|	$(\phi \wedge \phi)$	[logical conjunction]
	|	$(\phi \vee \phi)$	[logical disjunction]
	|	$(\phi \Rightarrow \phi)$	[logical implication]
	|	$(\mathbf{X}\,\phi)$	[ne**X**t state]
	|	$(\mathbf{F}\,\phi)$	[some **F**uture state]
	|	$(\mathbf{G}\,\phi)$	[all future states (**G**lobally)]
	|	$(\phi\,\mathbf{U}\,\phi)$	[**U**ntil]
	|	$(\phi\,\mathbf{W}\,\phi)$	[**W**eak-until]
	|	$(\phi\,\mathbf{R}\,\phi)$	[**R**elease]



Interpreting a Formula: Parse Trees (3)

ϕ	::=	$\top$	[*true*]
	|	$\bot$	[*false*]
	|	p	[propositional atom]
	|	$(\neg\phi)$	[logical negation]
	|	$(\phi \wedge \phi)$	[logical conjunction]
	|	$(\phi \vee \phi)$	[logical disjunction]
	|	$(\phi \Rightarrow \phi)$	[logical implication]
	|	$(\mathbf{X}\,\phi)$	[ne**X**t state]
	|	$(\mathbf{F}\,\phi)$	[some **F**uture state]
	|	$(\mathbf{G}\,\phi)$	[all future states (**G**lobally)]
	|	$(\phi\,\mathbf{U}\,\phi)$	[**U**ntil]
	|	$(\phi\,\mathbf{W}\,\phi)$	[**W**eak-until]
	|	$(\phi\,\mathbf{R}\,\phi)$	[**R**elease]

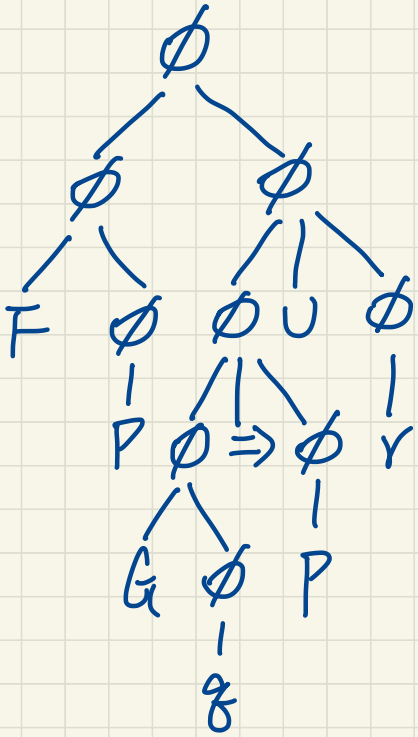
F p ∧ (G q ⇒ p U r)



Interpreting a Formula: Parse Trees (4)

ϕ	::=	$\top$	[*true*]
	|	$\bot$	[*false*]
	|	p	[propositional atom]
	|	$(\neg\phi)$	[logical negation]
	|	$(\phi \wedge \phi)$	[logical conjunction]
	|	$(\phi \vee \phi)$	[logical disjunction]
	|	$(\phi \Rightarrow \phi)$	[logical implication]
	|	$(\mathbf{X}\phi)$	[ne**X**t state]
	|	$(\mathbf{F}\phi)$	[some **F**uture state]
	|	$(\mathbf{G}\phi)$	[all future states (**G**lobally)]
	|	$(\phi \mathbf{U} \phi)$	[**U**ntil]
	|	$(\phi \mathbf{W} \phi)$	[**W**eak-until]
	|	$(\phi \mathbf{R} \phi)$	[**R**elease]

$$\mathbf{F}\ p \wedge ((\mathbf{G}\ q \Rightarrow p)\ \mathbf{U}\ r)$$



Interpreting a Formula: LMD (1)

ϕ	::=	$\top$	[*true*]
	|	$\bot$	[*false*]
	|	p	[propositional atom]
	|	$(\neg\phi)$	[logical negation]
	|	$(\phi \wedge \phi)$	[logical conjunction]
	|	$(\phi \vee \phi)$	[logical disjunction]
	|	$(\phi \Rightarrow \phi)$	[logical implication]
	|	$(\mathbf{X}\phi)$	[ne**X**t state]
	|	$(\mathbf{F}\phi)$	[some **F**uture state]
	|	$(\mathbf{G}\phi)$	[all future states (**G**lobally)]
	|	$(\phi \mathbf{U} \phi)$	[**U**ntil]
	|	$(\phi \mathbf{W} \phi)$	[**W**eak-until]
	|	$(\phi \mathbf{R} \phi)$	[**R**elease]

$\mathbf{F}\,p \wedge \mathbf{G}\,q \Rightarrow p\ \mathbf{U}\ r$

LMD

ϕ → left-most non-terminal

① $\phi \Rightarrow \phi$ — partially completed formula after 1st derivation

is derived to

② $\phi \wedge \phi \Rightarrow \phi$

③ $\mathbf{F}\phi \wedge \phi \Rightarrow \phi$

④ $\mathbf{F}p \wedge \phi \Rightarrow \phi$

⑤ $\mathbf{F}p \wedge \mathbf{G}\phi \Rightarrow \phi$

⑥ $\mathbf{F}p \wedge \mathbf{G}q \Rightarrow \phi$

⑦ $\mathbf{F}p \wedge \mathbf{G}q \Rightarrow \phi\ \mathbf{U}\ \phi$

⑧ $\mathbf{F}p \wedge \mathbf{G}q \Rightarrow p\ \mathbf{U}\ \phi$

⑨ $\mathbf{F}p \wedge \mathbf{G}q \Rightarrow p\ \mathbf{U}\ r$

Interpreting a Formula: LMD (2)

$$\mathbf{F}\,(p \wedge \mathbf{G}\,q \Rightarrow p\ \mathbf{U}\ r)$$

ϕ	::=	$\top$	[*true*]
	|	$\bot$	[*false*]
	|	p	[propositional atom]
	|	$(\neg\phi)$	[logical negation]
	|	$(\phi \wedge \phi)$	[logical conjunction]
	|	$(\phi \vee \phi)$	[logical disjunction]
	|	$(\phi \Rightarrow \phi)$	[logical implication]
	|	$(\mathbf{X}\,\phi)$	[ne**X**t state]
	|	$(\mathbf{F}\,\phi)$	[some **F**uture state]
	|	$(\mathbf{G}\,\phi)$	[all future states (**G**lobally)]
	|	$(\phi\ \mathbf{U}\ \phi)$	[**U**ntil]
	|	$(\phi\ \mathbf{W}\ \phi)$	[**W**eak-until]
	|	$(\phi\ \mathbf{R}\ \phi)$	[**R**elease]

implicit parentheses

① ϕ

② $\mathbf{F}(\phi)$

③ $\mathbf{F}(\phi \Rightarrow \phi)$

④ $\mathbf{F}(\phi \wedge \phi \Rightarrow \phi)$

⑤ $\mathbf{F}(p \wedge \phi \Rightarrow \phi)$

⑥ $\mathbf{F}(p \wedge \mathbf{G}\,\phi \Rightarrow \phi)$

⑦ $\mathbf{F}(p \wedge \mathbf{G}\,q \Rightarrow \phi)$

⑧ $\mathbf{F}(p \wedge \mathbf{G}\,q \Rightarrow \phi\ \mathbf{U}\ \phi)$

⑨ $\mathbf{F}(p \wedge \mathbf{G}\,q \Rightarrow p\ \mathbf{U}\ \phi)$

$\mathbf{F}(p \wedge \mathbf{G}\,q \Rightarrow p\ \mathbf{U}\ r)$

Interpreting a Formula: LMD (3)

ϕ	::=	$\top$	[*true*]
	|	$\bot$	[*false*]
	|	p	[propositional atom]
	|	$(\neg\phi)$	[logical negation]
	|	$(\phi \wedge \phi)$	[logical conjunction]
	|	$(\phi \vee \phi)$	[logical disjunction]
	|	$(\phi \Rightarrow \phi)$	[logical implication]
	|	$(\mathbf{X}\phi)$	[ne**X**t state]
	|	$(\mathbf{F}\phi)$	[some **F**uture state]
	|	$(\mathbf{G}\phi)$	[all future states (**G**lobally)]
	|	$(\phi\,\mathbf{U}\,\phi)$	[**U**ntil]
	|	$(\phi\,\mathbf{W}\,\phi)$	[**W**eak-until]
	|	$(\phi\,\mathbf{R}\,\phi)$	[**R**elease]

$$\mathbf{F}\ p \wedge (\mathbf{G}\ q \Rightarrow p\ \mathbf{U}\ r)$$

ϕ

① $\Rightarrow \phi \wedge \phi$

② $\Rightarrow \mathbf{F}\,\phi \wedge \phi$

③ $\Rightarrow \mathbf{F}\,p \wedge \phi$

④ $\Rightarrow \mathbf{F}\,p \wedge (\phi \Rightarrow \phi)$

⑤ $\Rightarrow \mathbf{F}\,p \wedge (\mathbf{G}\,\phi \Rightarrow \phi)$

⑥ $\Rightarrow \mathbf{F}\,p \wedge (\mathbf{G}\,q \Rightarrow \phi)$

⑦ $\Rightarrow \mathbf{F}\,p \wedge (\mathbf{G}\,q \Rightarrow \phi\ \mathbf{U}\ \phi)$

⑧ $\Rightarrow \mathbf{F}\,p \wedge (\mathbf{G}\,q \Rightarrow p\ \mathbf{U}\ \phi)$

⑨ $\Rightarrow \mathbf{F}\,p \wedge (\mathbf{G}\,q \Rightarrow p\ \mathbf{U}\ r)$

Interpreting a Formula: LMD (4)

$$\phi ::= \top \mid \bot \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \Rightarrow \phi) \mid (\mathbf{X}\phi) \mid (\mathbf{F}\phi) \mid (\mathbf{G}\phi) \mid (\phi\,\mathbf{U}\,\phi) \mid (\phi\,\mathbf{W}\,\phi) \mid (\phi\,\mathbf{R}\,\phi)$$

- $\top$ [*true*]
- $\bot$ [*false*]
- p [propositional atom]
- $(\neg\phi)$ [logical negation]
- $(\phi \wedge \phi)$ [logical conjunction]
- $(\phi \vee \phi)$ [logical disjunction]
- $(\phi \Rightarrow \phi)$ [logical implication]
- $(\mathbf{X}\phi)$ [neXt state]
- $(\mathbf{F}\phi)$ [some Future state]
- $(\mathbf{G}\phi)$ [all future states (Globally)]
- $(\phi\,\mathbf{U}\,\phi)$ [Until]
- $(\phi\,\mathbf{W}\,\phi)$ [Weak-until]
- $(\phi\,\mathbf{R}\,\phi)$ [Release]

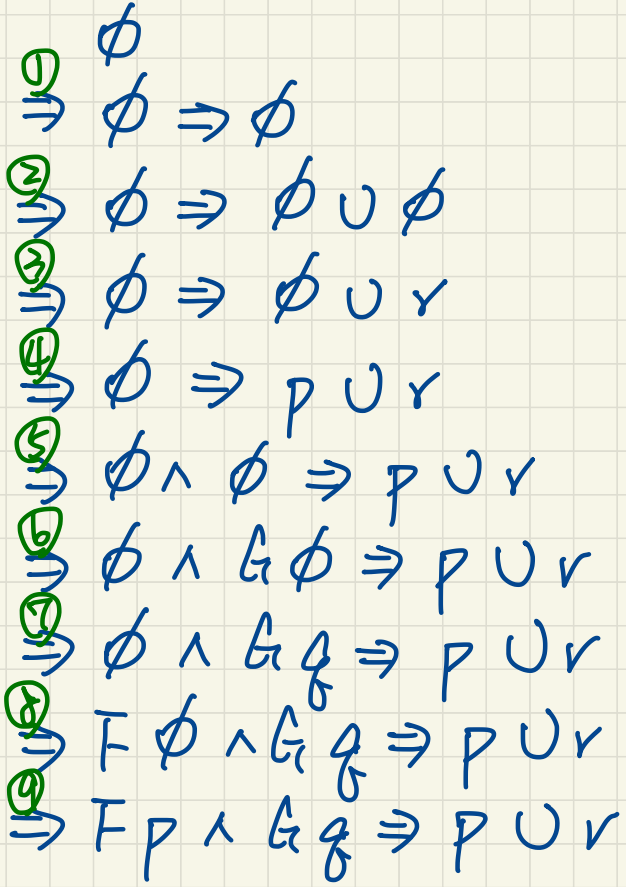
$$\mathbf{F}\ p \wedge ((\mathbf{G}\ q \Rightarrow p)\ \mathbf{U}\ r)$$

1. ϕ
2. $\Rightarrow \phi \wedge \phi$
3. $\Rightarrow \mathbf{F}\,\phi \wedge \phi$
4. $\Rightarrow \mathbf{F}\,p \wedge \phi$
5. $\Rightarrow \mathbf{F}\,p \wedge (\phi\ \mathbf{U}\ \phi)$
6. $\Rightarrow \mathbf{F}\,p \wedge ((\phi \Rightarrow \phi)\ \mathbf{U}\ \phi)$
7. $\Rightarrow \mathbf{F}\,p \wedge ((\mathbf{G}\,\phi \Rightarrow \phi)\ \mathbf{U}\ \phi)$
8. $\Rightarrow \mathbf{F}\,p \wedge ((\mathbf{G}\,q \Rightarrow \phi)\ \mathbf{U}\ \phi)$
9. $\Rightarrow \mathbf{F}\,p \wedge ((\mathbf{G}\,q \Rightarrow p)\ \mathbf{U}\ \phi)$
10. $\Rightarrow \mathbf{F}\,p \wedge ((\mathbf{G}\,q \Rightarrow p)\ \mathbf{U}\ r)$

Interpreting a Formula: RMD (1)

F p ∧ G q ⇒ p U r

ϕ	::=	$\top$	[*true*]
	|	$\bot$	[*false*]
	|	p	[propositional atom]
	|	$(\neg\phi)$	[logical negation]
	|	$(\phi \wedge \phi)$	[logical conjunction]
	|	$(\phi \vee \phi)$	[logical disjunction]
	|	$(\phi \Rightarrow \phi)$	[logical implication]
	|	$(\mathbf{X}\phi)$	[ne**X**t state]
	|	$(\mathbf{F}\phi)$	[some **F**uture state]
	|	$(\mathbf{G}\phi)$	[all future states (**G**lobally)]
	|	$(\phi \mathbf{U} \phi)$	[**U**ntil]
	|	$(\phi \mathbf{W} \phi)$	[**W**eak-until]
	|	$(\phi \mathbf{R} \phi)$	[**R**elease]



Interpreting a Formula: RMD (2)

$\mathbf{F}\,(p \wedge \mathbf{G}\,q \Rightarrow p\ \mathbf{U}\ r)$

ϕ	::=	$\top$	[*true*]
	|	$\bot$	[*false*]
	|	p	[propositional atom]
	|	$(\neg\phi)$	[logical negation]
	|	$(\phi \wedge \phi)$	[logical conjunction]
	|	$(\phi \vee \phi)$	[logical disjunction]
	|	$(\phi \Rightarrow \phi)$	[logical implication]
	|	$(\mathbf{X}\,\phi)$	[ne**X**t state]
	|	$(\mathbf{F}\,\phi)$	[some **F**uture state]
	|	$(\mathbf{G}\,\phi)$	[all future states (**G**lobally)]
	|	$(\phi\,\mathbf{U}\,\phi)$	[**U**ntil]
	|	$(\phi\,\mathbf{W}\,\phi)$	[**W**eak-until]
	|	$(\phi\,\mathbf{R}\,\phi)$	[**R**elease]

ϕ

① $\Rightarrow F(\phi)$

② $\Rightarrow F(\phi \Rightarrow \phi)$

③ $\Rightarrow F(\phi \Rightarrow \phi \cup \phi)$

④ $\Rightarrow F(\phi \Rightarrow \phi \cup r)$

⑤ $\Rightarrow F(\phi \Rightarrow p \cup r)$

⑥ $\Rightarrow F(\phi \wedge \phi \Rightarrow p \cup r)$

⑦ $\Rightarrow F(\phi \wedge G\phi \Rightarrow p \cup r)$

⑧ $\Rightarrow F(\phi \wedge Gq \Rightarrow p \cup r)$

⑨ $\Rightarrow F(p \wedge Gq \Rightarrow p \cup r)$

Interpreting a Formula: RMD (3)

$$\mathbf{F}\ p \wedge (\mathbf{G}\ q \Rightarrow p\ \mathbf{U}\ r)$$

ϕ	::=	$\top$	[*true*]
	|	$\bot$	[*false*]
	|	p	[propositional atom]
	|	$(\neg\phi)$	[logical negation]
	|	$(\phi \wedge \phi)$	[logical conjunction]
	|	$(\phi \vee \phi)$	[logical disjunction]
	|	$(\phi \Rightarrow \phi)$	[logical implication]
	|	$(\mathbf{X}\phi)$	[ne**X**t state]
	|	$(\mathbf{F}\phi)$	[some **F**uture state]
	|	$(\mathbf{G}\phi)$	[all future states (**G**lobally)]
	|	$(\phi\ \mathbf{U}\ \phi)$	[**U**ntil]
	|	$(\phi\ \mathbf{W}\ \phi)$	[**W**eak-until]
	|	$(\phi\ \mathbf{R}\ \phi)$	[**R**elease]

ϕ

① $\Rightarrow \phi \wedge \phi$

② $\Rightarrow \phi \wedge (\phi \Rightarrow \phi)$

③ $\Rightarrow \phi \wedge (\phi \Rightarrow \phi \cup \phi)$

④ $\Rightarrow \phi \wedge (\phi \Rightarrow \phi \cup r)$

⑤ $\Rightarrow \phi \wedge (\phi \Rightarrow p \cup r)$

⑥ $\Rightarrow \phi \wedge (G\phi \Rightarrow p \cup r)$

⑦ $\Rightarrow \phi \wedge (G\,q \Rightarrow p \cup r)$

⑧ $\Rightarrow F\phi \wedge (G\,q \Rightarrow p \cup r)$

⑨ $\Rightarrow F\,p \wedge (G\,q \Rightarrow p \cup r)$

Interpreting a Formula: RMD (4)

$F\ p \wedge ((G\ q \Rightarrow p)\ U\ r)$

ϕ	::=	$\top$	[*true*]
	|	$\bot$	[*false*]
	|	p	[propositional atom]
	|	$(\neg\phi)$	[logical negation]
	|	$(\phi \wedge \phi)$	[logical conjunction]
	|	$(\phi \vee \phi)$	[logical disjunction]
	|	$(\phi \Rightarrow \phi)$	[logical implication]
	|	$(\mathbf{X}\phi)$	[ne**X**t state]
	|	$(\mathbf{F}\phi)$	[some **F**uture state]
	|	$(\mathbf{G}\phi)$	[all future states (**G**lobally)]
	|	$(\phi\ \mathbf{U}\ \phi)$	[**U**ntil]
	|	$(\phi\ \mathbf{W}\ \phi)$	[**W**eak-until]
	|	$(\phi\ \mathbf{R}\ \phi)$	[**R**elease]

ϕ

① $\Rightarrow \phi \wedge (\phi)$

② $\Rightarrow \phi \wedge (\phi\ U\ \phi)$

③ $\Rightarrow \phi \wedge (\phi\ U\ r)$

④ $\Rightarrow \phi \wedge ((\phi \Rightarrow \phi)\ U\ r)$

⑤ $\Rightarrow \phi \wedge ((\phi \Rightarrow p)\ U\ r)$

⑥ $\Rightarrow \phi \wedge ((G\ \phi \Rightarrow p)\ U\ r)$

⑦ $\Rightarrow \phi \wedge ((G\ q) \Rightarrow p)\ U\ r)$

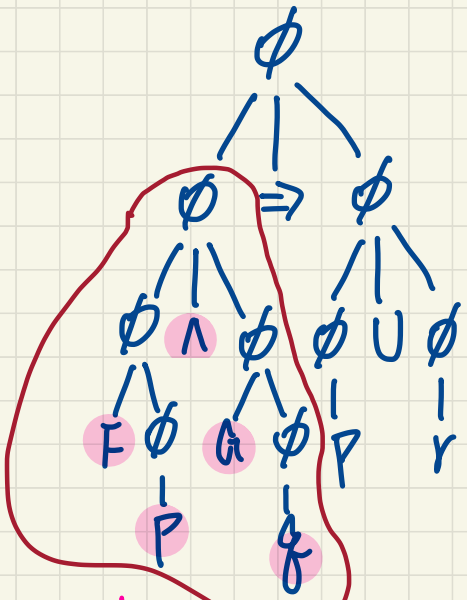
⑧ $\Rightarrow F\phi \wedge ((G\ q) \Rightarrow p)\ U\ r)$

⑨ $\Rightarrow F\ p \wedge ((G\ q) \Rightarrow p)\ U\ r)$

Interpreting a Formula: PT vs. LMD vs. RMD

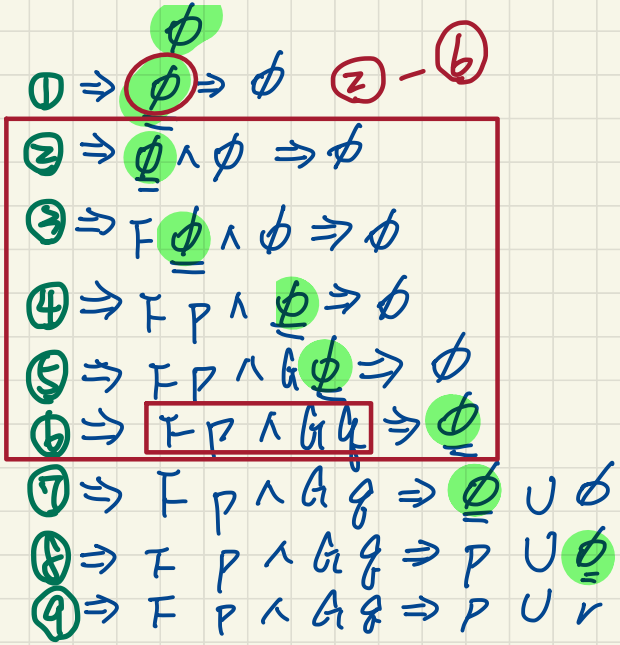
$\mathbf{F\,p \wedge G\,q \Rightarrow p\,U\,r} \in L(g)$

Parse Tree

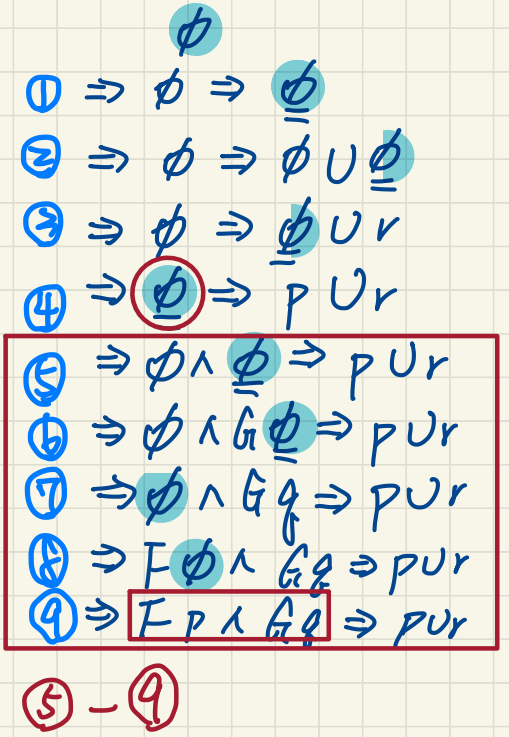


subformula/subtree: $Fp \wedge Gq$

LMD



RMD



Deriving Subformulas from a Parse Tree

Enumerate all subformulas of:

$$\mathbf{F}\ (p \Rightarrow \mathbf{G}\ r) \vee ((\neg\ q)\ \mathbf{U}\ p)$$

PT.

How many subtrees in PT?

10 subtrees